7.8 Present Value of an Ordinary Annuity

Owen and Anna are approaching retirement and are putting their finances in order. They have worked hard and invested their earnings so that they now have a large amount of money on which to live. They hire a financial advisor, and together they consider whether Owen and Anna have enough money to allow them to live comfortably for the rest of their lives by making **regular withdrawals** from an account.
To do this, they calculate the **present value** of an annuity based on Owen and Anna's projected living expenses.

1) The amount that must be invested NOW (you have the money) to provide payments of a specific amount at regular intervals over a certain term.

2) The amount borrowed or financed NOW that must be paid back by deposits.

Example 1:

Holly must begin to repay her student loan. Her monthly payments of $200 will be withdrawn at the end of each month from an account earning 6% interest compounded monthly.
How much must she deposit in the account today so that loan payments can be made for one year?

Payments

Method 1: Use a time line diagram

End of Month Payment	Calculations	Total
1	$200(1+0.06/12)^{-1}$	199
2	$200(1+0.06/12)^{-2}$	198.01
3	$200(1+0.06/12)^{-3}$	197.03
4	$200(1+0.06/12)^{-4}$	196.05
5	$200(1+0.06/12)^{-5}$	195.07
6	$200(1+0.06/12)^{-6}$	194.1
7	$200(1+0.06/12)^{-7}$	193.14
8	$200(1+0.06/12)^{-8}$	192.18
9	$200(1+0.06/12)^{-9}$	191.22
10	$200(1+0.06/12)^{-10}$	190.27
11	$200(1+0.06/12)^{-11}$	189.32
12	$200(1+0.06/12)^{-12}$	188.38
		2323.77

*** Notice ***

This should be less than total withdrawals (12 x 200 = 2400) because the money will be earning interest until payments are made

Method 2: Use a formula

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

where PV is the present value in $
R is the regular payment
i is interest rate per period
n is total the number of deposits

$$= \frac{200\left(1-\left(1+\frac{0.06}{12}\right)^{-12}\right)}{\frac{0.06}{12}}$$

$$= 2323.79$$

∴ she should deposit $2323.79 now.

Example 1:

Tommy Littlefinger has set up an annuity to help his son living expenses over the next 5 years. The annuity will pay $50 a month. The first payment will be made 1 month from now. The annuity earns 7.75%/a compounded monthly. How much money did Tommy put in the annuity?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$$= \frac{50\left(1-\left(1+\frac{0.0775}{12}\right)^{-60}\right)}{\left(\frac{0.0775}{12}\right)}$$

$$= 2480.53$$

∴ Tommy put $2480.53 into the annuity

Example 1:

Robin bought a bicycle for $1500. She arranged to make a payment to the store at the end of every month for a year. The store is charging 11%/a interest compounded monthly.
a) How much is each monthly payment? ✓
b) How much interest is Robin paying?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

a) $$1500 = \frac{R\left[1-\left(1+\frac{0.11}{12}\right)^{-12}\right]}{\left(\frac{0.11}{12}\right)}$$

$$1500 \times \left(\frac{0.11}{12}\right) = R[0.10371]$$

$$\frac{13.75}{0.1037} = \frac{R[0.1037]}{0.1037}$$

$$R = 132.58$$

b) $= 132.58 \times 12$

$= 1590.96$

HMWK:

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